AP Calculus AB

Unit 8 – Related Rates

AP Calculus AB - Worksheet 67 **Related Rates #1**

1)	Find $\frac{dS}{dt}$ if $S = 4\pi r^2$.
2)	Find $\frac{dV}{dt}$ if $V = \frac{1}{3}\pi r^2 h$.
3)	Find $\frac{dz}{dt}$ if $x^2 + y^2 = z^2$.
4)	If $x = 4,\ y = 3,\ \frac{dy}{dt} = -2,$ and $\frac{dx}{dt} = 0,$ then find $\frac{dz}{dt}$ for $x^2 + y^2 = z^2$.
5)	If $V = \frac{4}{3}\pi r^3$ and $\frac{dV}{dt} = 3\pi$ and $r = 6,$ then find $\frac{dr}{dt}$.
6)	If one leg AB of a right triangle increases at the rate of 2 in/sec, while the other leg AC decreases at 3 in/sec. Find how fast the hypotenuse is changing when AB is 6 ft. and AC is 8 ft.
7)	The volume of a cube is increasing at the rate of 1200 cm^3/min at the instant its edges are 20 cm long. At what rate are the edges changing at that instant?
8)	Each side of a square is increasing at a rate of 6 cm/sec. At what rate is the area of the square increasing when the area of the square is 16 cm^2?
9)	The radius of a sphere is decreasing at a rate of 2 $\frac{\text{mm}}{\text{sec}}$. How fast is the volume decreasing when the diameter is 80 mm?

Answers

1) $\frac{dS}{dt} = 8\pi r\frac{dr}{dt}$	2) $\frac{dV}{dt} = \frac{1}{3}\pi r^2\frac{dh}{dt} + \frac{2}{3}\pi rh\frac{dr}{dt}$	3) $\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$
4) $\frac{dz}{dt} = -\frac{6}{5}$	5) $\frac{dr}{dt} = \frac{1}{48}$	6) The hypotenuse is changing at a rate of $-\frac{1}{10}\frac{\text{ft}}{\text{sec}}$.
7) $\frac{ds}{dt} = 1\ \text{cm}/\text{min}$	8) $\frac{dA}{dt} = 48\ \text{cm}^3/\text{min}$	9) $\frac{dV}{dt} = -12800\pi\ \text{mm}^3/\text{sec}$

AP Calculus AB – Worksheet 68 **Related Rates 2**

1	Oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2 ft/sec. How fast is the area of the spill increasing when the radius is 60 feet?
2	A 26-foot ladder leans against a building so that the bottom moves away from the building at the rate of $3\ \frac{\text{ft}}{\text{sec}}$. When the foot of the ladder is 10 feet from the building, the top of is moving down at the rate of $r\ \frac{\text{ft}}{\text{sec}}$, where *r* is (A) $\frac{46}{3}$ (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) $\frac{5}{2}$ (E) $\frac{4}{5}$
3	A 13-foot plank is leaning against a wall. If, at a certain instant, the bottom of the plank is 5 ft. from the wall and is being pushed toward the wall at a rate of $6\frac{\text{in}}{\text{sec}}$, how fast, in feet per second is the top of the ladder moving up the wall?
4	The height of a rectangular box is 10 inches. Its length increases at the rate of $2\frac{\text{in}}{\text{sec}}$; its width decreases at the rate of $4\frac{\text{in}}{\text{sec}}$. When the length is 8 inches and the width 6 inches, the volume of the box is changing, in cubic inches per second, at the rate of (A) 200 (B) 80(C) -80 (D) -200 (E) -20
5	The radius of a sphere is decreasing at a rate of $2\ \frac{\text{mm}}{\text{sec}}$. How fast is the volume decreasing when the diameter is 80 mm?
6	Bike A is 45 miles west of an intersection and moving west at 20 miles/hour. Bike B is 15 miles south of the same intersection and moving north at 15 miles/hour. Are the bikes approaching each other or separating from each other and at what rate?
7	Show that $f\left(x\right)=\cos x-x$ has a zero in the interval $\left(0,1\right)$.
8	$\lim\limits_{x\to 5}\frac{x^3-125}{x-5}$

Answers

1) The area of the spill is increasing at a rate of $240\pi\ \frac{\text{ft}^2}{\text{sec}}$	2) C	3) $\frac{5}{24}\frac{\text{ft}}{\text{sec}}$	4) D
5) -12,800 $\pi\ \frac{\text{mm}^3}{\text{sec}}$	6) The two bikes are separating at $\frac{9\sqrt{10}}{2}$ miles per hour.	7)	8) 75

AP Calculus AB - Worksheet 69 **Related Rates #3**

1)	A spherical balloon is inflated at the rate of $100\frac{\text{ft}^3}{\text{min}}$. How fast is the balloon's radius increasing at the instant the radius is 5 feet?
2)	If a snowball melts so that its surface area decreases at a rate of $1\ \frac{\text{cm}^2}{\text{min}}$, find the rate at which the volume decreases when the diameter is 10 cm.
3)	The radius *r* of a right circular cone of fixed height of 20 cm is increasing at a rate of 2 cm/sec. How fast is the volume of the cone increasing when the radius is 10 cm?
4)	A boy is flying a kite. The kite is at a height of 150 feet. If the kite moves horizontally away from the boy at 20 ft/sec, how fast is the string being pulled out when its length is 250 feet?
5)	A circular conical reservoir has a depth of 20 feet and the radius of the top is 10 feet. Water is leaking out so the surface is falling at the rate of $\frac{1}{2}\ \frac{\text{ft}}{\text{hr}}$. The rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 feet deep is (A) 4π (B) 8π (C) 16π (D) $\frac{1}{4\pi}$ (E) $\frac{1}{8\pi}$
6)	A balloon is being filled with helium at the rate of 4 ft^3/min. Find the rate, in square feet per minute, at which the surface area is increasing when the volume is $\frac{32\pi}{3}$ cubic feet.

Answers

1) $\frac{dr}{dt}=\frac{1}{\pi}\frac{\text{ft}}{\text{min}}$	2) The volume of the snowball is decreasing at a rate of $\frac{5}{2}\frac{\text{cm}^3}{\text{min}}$.	3) $\frac{dV}{dt}=\frac{800\pi}{3}\ \text{cm}^3/\text{sec}$
4) The string is being pulled out at 16 ft/sec.	5) B	6) $\frac{dS}{dt}=4\ \frac{\text{ft}^2}{\text{min}}$

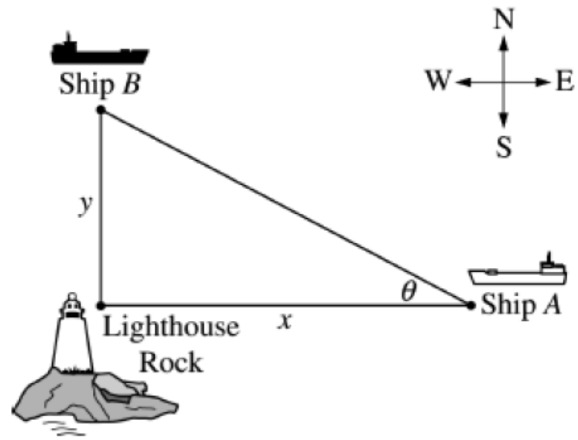
AP Calculus AB – Worksheet 70 **Related Rates #4**

1	A street light is mounted at the top of a 15-foot tall pole. A man 6 feet tall walks away from the pole with a speed of 5 ft/sec along a straight path. How fast is the tip of his shadow moving when he is 40 feet from the pole?
2	A liquid is flowing into a vertical cylindrical tank of radius 6 ft. at a rate of 8 ft^3/min. How fast is the top of the surface of the liquid rising?
3	The diameter and height of a paper cup in the shape of a cone are both 4 inches. Water is leaking out at the rate of $\frac{1}{2}\frac{\text{in}^3}{\text{sec}}$. Find the rate at which the water level is dropping when the <u>diameter</u> of the surface is 2 inches.
4	A spotlight on the ground shines on a wall 12 meters away. If a man 2 meters tall walks from the spotlight toward the building at a speed of 1.6 m/sec, how fast is the length of his shadow on the building decreasing when he is 4 meters from the building?
5	A kite 100 feet above the ground moves horizontally at a speed of 8 ft/sec. At what rate is the angle between the string and the horizontal decreasing when 200 feet of string has been let out?
6	A block of ice in the form of a cube has one edge 10 ft. long. It is melting so that its dimensions decrease at the rate of 1/10 ft per second. At what rate is the volume of the block decreasing when the edge is 5 ft. long? The block always remains in the shape of a cube.
7	The radius of a circular metal plate is increasing at the rate of 0.01 cm per second when heated. At what rate is the area of the plate increasing when the radius is 5 cm?

Answers

1) $\frac{25}{3}\frac{\text{ft}}{\text{sec}}$	2) The surface of the liquid is rising at a rate of $\frac{2}{9\pi}\frac{\text{ft}}{\text{min}}$	3) Water is level is dropping at $\frac{1}{2\pi}\frac{\text{in}}{\text{sec}}$.
4) The man's shadow is decreasing in length at a rate of .6 m/sec.	5)	6) The volume of the cube is decreasing at a rate of 7.5 cubic feet per second.
7) The area of the plate is increase at a rate of $\frac{\pi}{10}$ square cm per second.		

(2002B AB6) Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure at right.

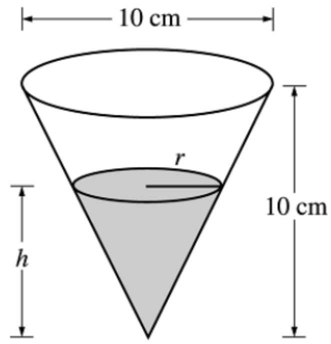


(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

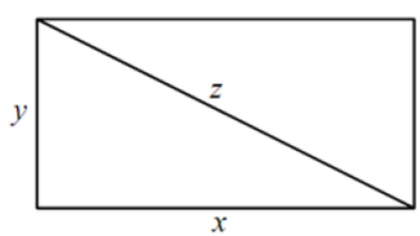
(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

(2002 AB5) A container has the shape of an open right circular cone, as shown in the figure below. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{3}{10}$ cm/hr.



(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.



The sides of the rectangle at right increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?